

## 問題

問題 1. 次の和を求めよ。

$$(1) S = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)}$$

$$(2) S = \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{5 \cdot 6} + \cdots + \frac{1}{(n+2)(n+3)}$$

$$(3) S = \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \cdots + \frac{1}{(3n-2)(3n+1)}$$

$$(4) S = \frac{1}{1 \cdot 5} + \frac{1}{5 \cdot 9} + \frac{1}{9 \cdot 13} + \cdots + \frac{1}{(4n-3)(4n+1)}$$

問題 2. 次の和を求めよ。

$$(1) S = 1 \cdot 1 + 2 \cdot 2 + 3 \cdot 2^2 + \cdots + n \cdot 2^{n-1}$$

$$(2) S = 1 \cdot 1 + 2 \cdot 4 + 3 \cdot 4^2 + \cdots + n \cdot 4^{n-1}$$

$$(3) S = 1 \cdot 1 + 3 \cdot 3 + 5 \cdot 3^2 + \cdots + (2n-1) \cdot 3^{n-1}$$

$$(4) S = 1 \cdot 1 + 4 \cdot 5 + 7 \cdot 5^2 + \cdots + (3n-2) \cdot 5^{n-1}$$

$$(5) S = 1 \cdot 1 + 2 \cdot \frac{1}{2} + 3 \cdot \left(\frac{1}{2}\right)^2 + \cdots + n \cdot \left(\frac{1}{2}\right)^{n-1}$$

$$(6) S = 1 \cdot 1 + 3 \cdot \frac{1}{3} + 5 \cdot \left(\frac{1}{3}\right)^2 + \cdots + (2n-1) \cdot \left(\frac{1}{3}\right)^{n-1}$$

## 練習

練習 1. 次の和を求めよ。

$$(1) S = \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \cdots + \frac{1}{(n+1)(n+2)}$$

$$(2) S = \frac{1}{4 \cdot 5} + \frac{1}{5 \cdot 6} + \frac{1}{6 \cdot 7} + \cdots + \frac{1}{(n+3)(n+4)}$$

$$(3) S = \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \cdots + \frac{1}{(2n-1)(2n+1)}$$

$$(4) S = \frac{1}{2 \cdot 5} + \frac{1}{5 \cdot 8} + \frac{1}{8 \cdot 11} + \cdots + \frac{1}{(3n-1)(3n+2)}$$

練習 2. 次の和を求めよ。

$$(1) S = 1 \cdot 1 + 2 \cdot 3 + 3 \cdot 3^2 + \cdots + n \cdot 3^{n-1}$$

$$(2) S = 1 \cdot 1 + 2 \cdot 5 + 3 \cdot 5^2 + \cdots + n \cdot 5^{n-1}$$

$$(3) S = 1 \cdot 1 + 4 \cdot 4 + 7 \cdot 4^2 + \cdots + (3n-2) \cdot 4^{n-1}$$

$$(4) S = 1 \cdot 1 + 5 \cdot 6 + 9 \cdot 6^2 + \cdots + (4n-3) \cdot 6^{n-1}$$

$$(5) S = 1 \cdot 1 + 2 \cdot \frac{1}{4} + 3 \cdot \left(\frac{1}{4}\right)^2 + \cdots + n \cdot \left(\frac{1}{4}\right)^{n-1}$$

$$(6) S = 1 \cdot 1 + 4 \cdot \frac{1}{3} + 7 \cdot \left(\frac{1}{3}\right)^2 + \cdots + (3n-2) \cdot \left(\frac{1}{3}\right)^{n-1}$$

## 解答

問題 1.

$$(1) S = \frac{n}{n+1} \quad (2) S = \frac{n}{3(n+3)} \quad (3) S = \frac{n}{3n+1} \quad (4) S = \frac{n}{4n+1}$$

問題 2.

$$(1) S = (n-1) \cdot 2^n + 1 \quad (2) S = \frac{(3n-1) \cdot 4^n + 1}{9} \quad (3) S = (n-1) \cdot 3^n + 1$$

$$(4) S = \frac{(12n-11) \cdot 5^n + 11}{16} \quad (5) S = 4 - (n+2) \cdot \left(\frac{1}{2}\right)^{n-1}$$

$$(6) S = 3 - (n+1) \cdot \left(\frac{1}{3}\right)^{n-1}$$

練習 1.

$$(1) S = \frac{n}{2(n+2)} \quad (2) S = \frac{n}{4(n+4)} \quad (3) S = \frac{n}{2n+1} \quad (4) S = \frac{n}{2(3n+2)}$$

練習 2.

$$(1) S = \frac{(2n-1) \cdot 3^n + 1}{4} \quad (2) S = \frac{(4n-1) \cdot 5^n + 1}{16} \quad (3) S = (n-1) \cdot 4^n + 1$$

$$(4) S = \frac{(20n-19) \cdot 6^n + 19}{25} \quad (5) S = \frac{16}{9} - \frac{3n+4}{9} \cdot \left(\frac{1}{4}\right)^{n-1}$$

$$(6) S = \frac{15}{4} - \frac{6n+5}{4} \cdot \left(\frac{1}{3}\right)^{n-1}$$