

問題

問題 1. 次の条件によって定められる数列 $\{a_n\}$ の一般項を求めよ。

$$(1) a_1 = \frac{1}{2}, \frac{1}{a_{n+1}} = \frac{2}{a_n} + 3$$

$$(2) a_1 = \frac{1}{3}, \frac{1}{a_{n+1}} = \frac{1}{a_n} + 4n$$

$$(3) a_1 = \frac{1}{3}, \frac{1}{a_{n+1}} = \frac{2a_n + 1}{a_n}$$

$$(4) a_1 = 1, \frac{1}{a_{n+1}} = \frac{4a_n + 3}{a_n}$$

$$(5) a_1 = 1, a_{n+1} = \frac{a_n}{3a_n + 4}$$

$$(6) a_1 = \frac{1}{2}, a_{n+1} = \frac{a_n}{4a_n + 1}$$

問題 2. 次の条件によって定められる数列 $\{a_n\}$ の一般項を求めよ。

$$(1) a_1 = 2, a_{n+1} = 2a_n + 2^n$$

$$(2) a_1 = 9, a_{n+1} = 2a_n + 3^n$$

$$(3) a_1 = 10, a_{n+1} = 10a_n + 2 \cdot 5^n$$

$$(4) a_1 = -4, a_{n+1} = 2a_n + (-2)^{n+2}$$

問題 3. 次の条件によって定められる数列 $\{a_n\}$ の一般項を求めよ。

$$(1) a_1 = 3, \frac{a_{n+1}}{n+1} = \frac{a_n}{n} + 2n$$

$$(2) a_1 = 2, \frac{a_{n+1}}{n+1} = \frac{2a_n}{n}$$

$$(3) a_1 = 3, na_{n+1} = 3(n+1)a_n + 2n(n+1)$$

$$(4) a_1 = 5, na_{n+1} = (n+1)a_n + 2n(n+1)$$

練習

練習 1. 次の条件によって定められる数列 $\{a_n\}$ の一般項を求めよ。

$$(1) a_1 = \frac{1}{5}, \frac{1}{a_{n+1}} = \frac{1}{a_n} + 2n$$

$$(2) a_1 = 1, \frac{1}{a_{n+1}} = \frac{1}{a_n} + 5$$

$$(3) a_1 = \frac{1}{3}, \frac{1}{a_{n+1}} = \frac{-2a_n + 1}{a_n}$$

$$(4) a_1 = \frac{1}{2}, \frac{1}{a_{n+1}} = \frac{6a_n + 3}{a_n}$$

$$(5) a_1 = 1, a_{n+1} = \frac{a_n}{4a_n + 2}$$

$$(6) a_1 = \frac{1}{4}, a_{n+1} = \frac{a_n}{3a_n - 2}$$

練習 2. 次の条件によって定められる数列 $\{a_n\}$ の一般項を求めよ。

$$(1) a_1 = 6, a_{n+1} = 3a_n + 3^n$$

$$(2) a_1 = 2, a_{n+1} = 3a_n + 2^n$$

$$(3) a_1 = 4, a_{n+1} = 2a_n + 3 \cdot 4^n$$

$$(4) a_1 = -3, a_{n+1} = -3a_n + 2 \cdot (-3)^{n+1}$$

練習 3. 次の条件によって定められる数列 $\{a_n\}$ の一般項を求めよ。

$$(1) a_1 = 2, \frac{a_{n+1}}{n+1} = \frac{a_n}{n} + 2n + 1$$

$$(2) a_1 = 3, \frac{a_{n+1}}{n+1} = \frac{2a_n}{n} + 5$$

$$(3) a_1 = 3, na_{n+1} = 3(n+1)a_n$$

$$(4) a_1 = 1, na_{n+1} = (n+1)a_n + 2n^2(n+1)$$

解答

問題 1.

$$(1) a_n = \frac{1}{5 \cdot 2^{n-1} - 3} \quad (2) a_n = \frac{1}{2n^2 - 2n + 3} \quad (3) a_n = \frac{1}{2n + 1}$$

$$(4) a_n = \frac{1}{3^n - 2} \quad (5) a_n = \frac{1}{2 \cdot 4^{n-1} - 1} \quad (6) a_n = \frac{1}{4n - 2}$$

問題 2.

$$(1) a_n = (n + 1) \cdot 2^{n-1} \quad (2) a_n = 3 \cdot 2^n + 3^n \quad (3) a_n = 5^{n-1} \cdot (3 \cdot 2^{n+1} - 2)$$

$$(4) a_n = -3 \cdot 2^n - (-2)^n$$

問題 3.

$$(1) a_n = n^3 - n^2 + 3n \quad (2) a_n = n \cdot 2^n \quad (3) a_n = n(4 \cdot 3^{n-1} - 1)$$

$$(4) a_n = 2n^2 + 3n$$

練習 1.

$$(1) a_n = \frac{1}{n^2 - n + 5} \quad (2) a_n = \frac{1}{5n - 4} \quad (3) a_n = -\frac{1}{2n - 5} \quad (4) a_n = \frac{1}{5 \cdot 3^{n-1} - 3}$$

$$(5) a_n = \frac{1}{5 \cdot 2^{n-1} - 4} \quad (6) a_n = \frac{1}{3 \cdot (-2)^{n-1} + 1}$$

練習 2.

$$(1) a_n = (n + 5) \cdot 3^{n-1} \quad (2) a_n = 4 \cdot 3^{n-1} - 2^n \quad (3) a_n = -2^n + 6 \cdot 4^{n-1}$$

$$(4) a_n = (2n + 1) \cdot (-3)^n$$

練習 3.

$$(1) a_n = n^3 + n \quad (2) a_n = n(2^{n+2} - 5) \quad (3) a_n = n \cdot 3^n$$

$$(4) a_n = n^3 - n^2 + n$$